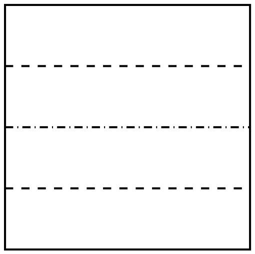
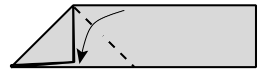
Pentagon-Hexagon Zig-Zag (PHiZZ) Units

Design by Tom Hull

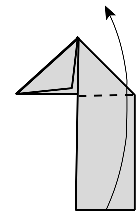


Accordian pleat the paper into fourths.

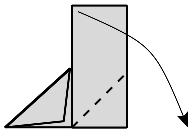
Fold top left corner down.



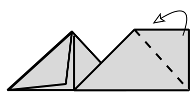
Fold the right edge of the strip down to meet the folded edge.



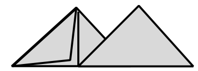
Fold the strip up, making the bottom flush.



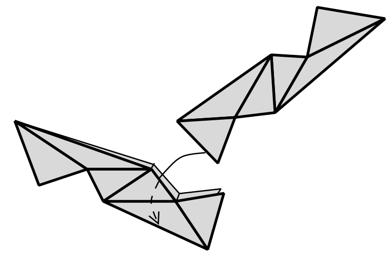
Fold the strip down to the right.



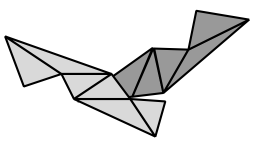
Fold the upper right corner down behind the unit.



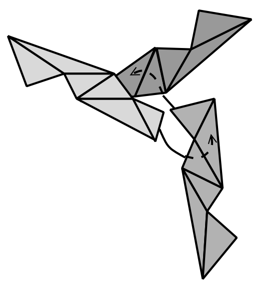
The completed unit.



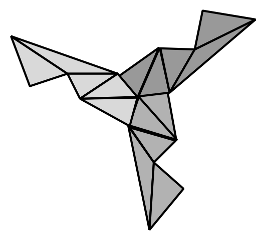
Insert one unit into the other one from the side so that the creases overlap.



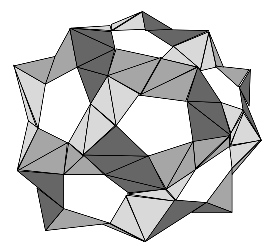
The two units are joined.



Insert a third unit into the second unit in the same manner. Tuck the first unit into the third unit.



The completed group of three units.



30 PHiZZ units make a dodecahedron. You can also use PHiZZ units to make larger buckyballs, tori, and other surfaces

Pentagons in a Buckyball

Buckyballs are closed, three-dimensional shapes with two properties: each face is a pentagon or a hexagon, and exactly three polygons meet at each vertex. Buckyballs satisfy Euler's formula: $V - E + F = 2$, where V stands for the number of vertices, E is the number of edges, and F is the number of faces.

Each edge in a buckyball touches two vertices. By definition, each vertex of a buckyball is shared by three edges. This means that we can relate the number of vertices to the number of edges using the formula $V = \frac{2E}{3}$. Substituting this into Euler's formula and simplifying gives us $-\frac{E}{3} + F = 2$.

Let P stand for the number of pentagons in the buckyball, and let H stand for the number of hexagons, so that $F = P + H$. Each pentagon is surrounded by 5 edges and each hexagon is surrounded by 6 edges. Each edge participates in two polygons. This gives us $E = \frac{5P+6H}{2}$. We can use this information to get everything in Euler's formula in terms of P and H: $-\frac{5P+6H}{6} + (P + H) = 2$.

In simplifying this expression, the H's cancel out and we end up with $P = 12$. The fact that the hexagons drop out of the equation is rather amazing. This calculation shows us that no matter how large the buckyball is, there will always be exactly 12 pentagons. There can be almost any number of hexagons.

Curvature in Surfaces Made From PHiZZ Units

A PHiZZ torus (donut-shape) contains several different types of polygons. In the models I created, there are many hexagons, some pentagons around the outside, and some heptagons in the inner ring. (It is possible to use squares or triangles instead of pentagons, and octagons or larger polygons instead of heptagons.) The polygons involved in the construction determine the curvature at each point of the surface.

Each angle of a regular hexagon measures 120 degrees and so angles from three hexagons meeting at a vertex total 360 degrees. This means that regular hexagons join together to form a flat surface with "zero curvature". Because a flat sheet can be rolled into a tube, hexagons can also join together to make a cylinder. Regular pentagons have angles measuring 108 degrees. When one pentagon meets two hexagons, the sum of the angles around the vertex is less than 360 degrees. This results in "positive curvature" like the surface of a ball. Regular heptagons and larger polygons have angles measuring more than 120 degrees. When one of these shapes meets two hexagons or when two of these shapes meet one hexagon, the angles around the vertex total more than 360 degrees. This causes the surface to buckle so that the surface curves up in one direction and down in another direction like a saddle. This is called "negative curvature".

A torus involves all three types of curvature. Around the outside of the torus, the surface has positive curvature like a ball. In the inner ring of the torus, the surface has negative curvature like a saddle. The rest of the torus has flat curvature like a cylinder.

Applications

These mathematical ideas have important applications in biology, chemistry, and physics. Many scientists and engineers are studying structures built from carbon atoms using the mathematical ideas outlined above. Buckminsterfullerene, C_{60}, is a 1-spherical buckyball made from carbon atoms. Carbon atoms can also be joined together to make nanotubes with different properties. Carbon structures of this type are very flexible and strong, they conduct heat without expanding, and they have interesting electrical properties. These structures may be used in the future to create new plastics, ceramics, thermal materials, fabrics, electronics at a molecular scale, biomedical materials, and air and water filtration systems.