Creating Polyhedra with Snapology

Faye E. Goldman
616 Valley View Road
Ardmore PA 10003
FayeG@ix.netcom.com

Abstract

This workshop provides an introduction to the Snapology Technique. This technique, invented by Heinz Strobl (Germany), makes beautiful polyhedra. Paper strips or ribbons are folded to form rectangular prisms which appear extruded from the edges of the base polyhedron. Once the technique is understood, it is possible to create an endless number and type of polyhedra. Participants will learn the basic technique and use it to make an icosahedron.

Introduction

There are many ways of making polyhedra using origami, the art of paper folding. When most people think about using origami, they think they are limited to one square piece of paper, no cuts or glue. Indeed, John Montroll [4,5,6] creates polyhedra using only a single sheet of square paper. Most origami designers use many identical modules to create each polyhedron. Modules can be made to represent faces, edges, or vertices. Tomoko Fuse has designed many different modules [1] representing edges, faces and vertices. The most famous unit was created by Mitsonobu Sonobe [3]. This unit can be used both as a face and edge unit.

Snapology is unique in that the technique uses strips to represent both faces and edges of polyhedra.

About the Snapology Technique

Heinz Strobl invented Snapology at the end of the 1990's to use up the left-over pieces of ticker tape he used in the development of 'Knotology' [7]. The Knotology Technique uses long paper strips to create pentagons composed of flat knots. Beautiful three-dimensional spherical models are created by varying how the adjacent knots are connected [8].

Strobl uses strips of paper, my choice of material is polypropylene ribbon.

There are two types of strip modules used in the Snapology Technique:

1) The **scaffold module** refers to the inside structure. It outlines the face of each of the polygons with a doubled wall of ribbon or paper. The width of the strip of paper or ribbon is one unit. The length of each scaffold module is 2*N, where N is the number of edges of the polygon. A triangle face uses a strip six units long, a square eight, etc.

2) The **hinge module** acts as an edge and connects two adjacent scaffold modules (faces). This workshop will cover 'Special Snapology' where all hinge modules are exactly four units long.

The Snapology Technique builds rectangular prisms on the surface of each face of the polyhedron. Adjacent rectangular prisms are connected with a hinge unit. This unit covers the prism, but spreads open and lets a bit of the underlying prism peek through. The more acute the angle between these prisms, the more stable the resulting Snapology model. In the case of the icosahedron, the angle between adjacent prisms is 42^o. This is calculated by taking the dihedral angle of the icosahedron (138^o) and subtracting from 180°. Models made using the Snapology Technique work best if the angle between adjacent prisms is less than 45°. If you remember your geometry, the icosahedron is one of the five Platonic solids, made of 20 equilateral triangles. There are 30 edges and 12 vertices. Each vertex is surrounded by 5 triangles. The icosahedron makes a good first project for Snapology (Figure 1).

Making the Icosahedron

Figure 1: *Icosahedron made from paper strips.*

There are multiple ways of creating a 1xN strip. I recommend making a "witch's ladder" (Figure 2) and cutting the strips apart. This method produces uniform strips with pre-marked squares.

How long should the strips be? As mentioned earlier, the icosahedron has 20 triangular faces and 30 edges. You will need 20 1x6 strips for the scaffold (faces) and 30 1x4 strips for the edges (hinges). The scaffold needs 20 x 6 = 120 squares and the hinges require 30 x 4 =120. (It is left for the reader to ponder if the equal lengths is a coincidence.) If ¾" ribbon is used, then 90" are needed for each type of strip. I recommend using a different color for the scaffold and hinge pieces. I suggest two 48" lengths for each color.

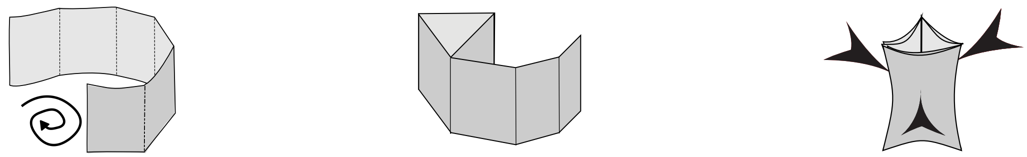
Take one strip of each color and place the ends perpendicular to each other. Take the lower strip and fold it over the upper strip, making the fold as close as possible to the upper strip. Repeat with the other strip, which is now the lower one. Continue this alternating motion until done. The result will be uniform folds along the length of each strip. The folds will alternate in direction.

Figure 2: Finished "witch's ladder".

It is important to make the creases uniform and crisp. When making the "witch's ladder", the result is more precise if you rotate 90° between folds and use the same motion for each fold. Using an identical motion will produce a better result than changing motions as made by left over, bottom up, right over etc.

When the ladders are complete, the folded strips are ready to be cut into the lengths needed for the project. Cut as close to the middle of the fold as possible. The color which will be most visible will be the hinge color. Cut 30 1x4 strips of this color. Cut the other color into 20 1 x 6 strips for the scaffold. Crease all the strips so that there are mountain creases on the same side. If one of the sides of the strips is prettier, the mountain creases should be on this side. It will be necessary to reverse some of the creases.

Figures 3-8 detail the beginning steps in making an icosahedron using the Snapology Technique.



(a) (b) (c)

Figure 3: *Wrap a 1x6 strip around itself to form a triangle with 2 layers (a) (b). Squeeze and sharpen the corners (c). (Create a triangle.)*

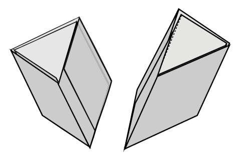
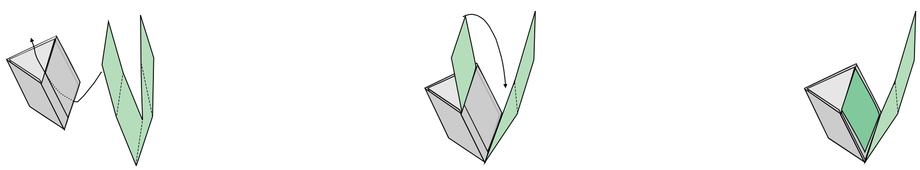
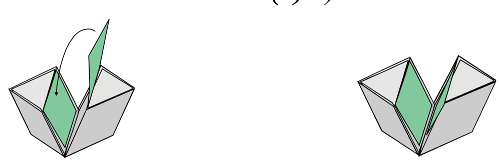


Figure 4: *Take another 1x6 strip and repeat Figure 3. Place the loose flaps facing each other.*



(a) (b) (c)

Figure 5: *Take a 1x4 strip of the hinge material (a). Feed the hinge material through one of the triangles so that the middle fold is at the bottom. Snap down the end of the hinge which is going through the triangle (b). (This combination of a hinge unit and a scaffold unit will be referred to as a "combination unit (c)".)*



(a) (b)

Figure 6: *Feed the open part of the hinge through the other triangle from Figure 4 pushing the triangle down as far as possible (a). Snap the open part of the hinge down between the two triangles (b). This snapping in place is how the Snapology Technique got its name.*

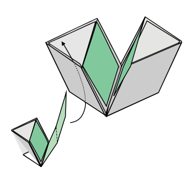


Figure 7: *Create a combination unit (see Figure 5.) and add to the existing model. Add 2 more combination units forming a ring around a point. When there are 5 triangles in a ring, connect the first and last triangles with a hinge unit.*

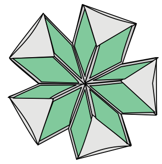
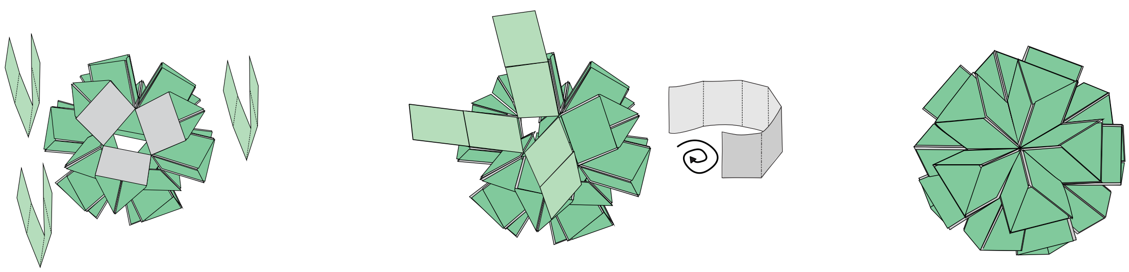


Figure 8: *Finished ring of 5. The model is now quite stable.*

Keep adding combination units, making rings of five triangles as quickly as possible. The ring of five triangles is more stable than loose triangles. When you choose the next place to attach a combination unit, choose a place where the ring of five triangles is more complete. If there are three triangles already completed, complete that ring with two combination units and an extra hinge before starting a new ring.

While building, make sure that each ring has exactly five triangles and not six or four triangles. This is a common mistake. The hinges must always be placed on the inside of the model. Miniature clothespins or paperclips can be used to hold the pieces together. It is necessary to remove the clips once a ring of 5 triangles is formed. The ring will spread out and look like a flower. Refer to Figure 8.

The last, or twentieth triangle is the most difficult as there is no longer any space behind the model in order to work. Complete this triangle by first placing half of each of the last three hinges. Then wrap the scaffold around twice to form a triangle. Finally snap each of the three hinges into place. Refer to Figure 9.



(a) (b) (c)

Figure 9: *Complete the last (20th) triangle by first placing half of each of the last 3 hinges in the unfinished triangles(a). Then wrap the scaffold around the pieces sticking out(b). Finally, 'snap' the other half of the hinges in place (c).*

(a) (b)

Figure 10: *Icosahedra made with different width strips. 3/8" scaffold and 3/4" hinge (a) 3/4" scaffold and 3/8" hinge (b).*

Regular and Semi Regular Convex Polyhedra											
	Polyhedron Name	Number of Vertices	Vertex Description	Type/Number of Polygons (Scaffolds) Triangles (3)	Quadralaterals (Squares) (4)	Pentagons (5)	Hexagons (6)	Octagons (8)	Decagons (10)	Number of edges (Hinges)	Hinge Type *
			Scaffold: Strip length:	6	8	10	12	16	20		
Platonic Solids:											
1	Tetrahedron	4	(3,3,3)	4						6	II
2	Octahedron	6	(3,3,3,3)	8						12	**I-II**
3	Hexahedron (cube)	8	(4,4,4)		6					12	**II**
4	Icosahedron	12	(3,3,3,3,3)	20						30	I
5	Dodecahedron	20	(5,5,5)			12				30	I
Archimedean Solids:											
6	Truncated tetrahedron	12	(3,6,6)	4			4			18	I
7	Truncated octahedron	24	(4,6,6)		6		8			36	I
8	Truncated cube (hexahedron)	24	(3,8,8)	8					6	36	**II**
9	Truncated icosahedron	60	(5,6,6)			12	20			90	I
10	Truncated dodecahedron	60	(3,10,10)	20					12	90	I
11	Cuboctahedron	12	(3,4,3,4)	8	6					24	I
11a	*Rhombic dodecahedron* ***(Dual)***	14			12					24	I
12	Icosidodecahedron	30	(3,5,3,5)	20		12				60	I
12a	*Rhombic triacontahedron* ***(Dual)***	32			30					60	I
13	(Small) Rhombi-cuboctahedron	24	(3,4,4,4)	8	18					48	I
14	Small Rhombicosi-dodecahedron	60	(3,4,5,4)	20	30	12				120	I
15	Truncated cuboctahedron/	48	(4,6,8)		12		8	6		72	I
16	Great Rhombicosidodeca-hedron/ Trunc. Icosidodecahedron	120	(4,6,10)		30		20		12	180	I
17	Snub cube	24	(3,3,3,3,4)	32	6					60	I
18	Snub dodecahedron	60	(3,3,3,3,5)	80		12				150	I

*Type I hinges are 1x4
Type II hinges are 1x6 (must be used with dihedral angle less than about 110°)
The *Rhombic dodecahedron* and *Rhombic triacontahedron* are listed with their Archimedean Duals because they work well with the Snapology Technique.

Table 1: *Platonic and Archimedean Solids*

Further Explorations

The Platonic and Archimedean Solids make beautiful models. I've included information on the Platonic and Archimedean Solids. **Table 1** has details to assist in making the polyhedra. A book/kit is available which provides instructions and paper strips to make 15 projects [2].

To explore further, it is necessary to understand the underlying structure of the model. If the dihedral angle between adjacent faces is less than 110º, such as a cube or (90º) or tetrahedron (~71º) then the 4-unit hinge won't hold in place without glue. A six-unit hinge where the first and last units slip under the scaffold will hold the hinge in place [9].

Another polyhedron which works well is the icosidodecahedron. Following line 12 in **Table 1**, you'll need 20 of 1x6 strips (triangles) and 12 1x10 (pentagons) for a total of 240 units for the scaffolds. 60 edges require 60 1x4 hinges for 240 units. Using ¾" ribbon requires 180" in each of two colors. The vertices consist of two pairs of alternating triangles and pentagons.

This workshop covers the basics of the Snapology Technique. At this level, hinge units consist of only four units, both hinge and scaffold strips are of equal width and all hinges are on the inside of the completed model. Each of these rules may be bent or broken to make more involved shapes. **Figure 10** shows icosahedra made with very different width ribbon. One is short and squat, the other long and slender. It will be left as an exercise for the reader to determine if the circumscribed spheres are identical.

It does require time and concentration to create models using this technique, but the end result is beautiful. Snapology is a wonderful way to take an abstract concept like polyhedra and realize it with beautiful concrete models.

References

[1] Fuse, Tomoko, *Unit Origami: Multidimensional Transformations*. Japan Publications. 1990.

[2] Goldman, Faye E. *Geometric Origami*, Thunder Bay Press. 2014.

[3] Lister, David, Origins of the Sonobe module, http://www.britishorigami.info/academic/lister/sonobe.php (as of February 26, 2017)

[4] Montroll, John, *3D Origami Platonic Solids & More*. 2012.

[5] Montroll, John, *A Constellation of Origami Polyhedra*. Dover. 2004.

[6] Montroll, John, *A Plethora of Polyhedra in Origami 2nd Revised Edition*. Antroll Publishing Company. 2015.

[7] Strobl, Heinz, *Personal communications*, 2010

[8] Strobl, Heinz, Knotology/Snapology, http://www.knotology.eu (as of February 26, 2017).

[9] Strobl, Heinz, Snapology: Connection module type 2 http://www.knotology.eu/PPP-Jena2010e/S24.html (as of February 26, 2017).